Real Options and Facilities Access Regulation

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Abstract

Real options achieve their value from flexible management response to signals about uncertainty. Any constraint on flexibility will obviously impair the real option value.

Regulation imposes constraints on operations, but also provides tariffs to the regulated entity. Thus, it is not obvious that the regulated entity necessarily suffers a value loss from regulation, if the tariffs are excessively generous. The question of whether the tariffs are excessively generous is a question of social optimality. We address this question in the context of facilities access, which is a popular method of "deregulating" industries that had monopoly power. The regulator determines tariffs for access to the production facilities of the oligopolist facility owner so that competitors can use the facilities and offer the consumption good in a competitive market. This is the basis for deregulation of power, gas distribution and telecom industries. It has also been proposed in industries that were not formerly regulated, such as the rail infrastructure for integrated mining industries.

Pindyck has suggested that access tariffs in these cases should include compensation for the capital costs plus the real option premium that was extinguished to establish the facility. But he offers no proof, nor any analysis of the magnitude and direction of the distortion if compensation is only offered for the capital costs only. Also, he does not consider two-part tariff structure with an up-front access tariff, plus an annual tariff for capacity. In this paper, we investigate these issues numerically.

1 Introduction

In this paper, we investigate optimal tariffs for facilities access when the facility Builder has a real option to choose the optimal timing of construction. Pindyck (2004, 2005) suggests that the cost base for a regulated access tariff should include actual out-of-pocket costs of the facilities Builder plus the value of real options that are extinguished by committing capital and building. Pindyck suggests that not allowing for a recovery of the real option value will distort the ex ante incentives of the facilities Builder and not result in a first-best solution. However, he focuses on the analysis of a one-part tariff that would recover the total cost (capital plus option opportunity cost) without analyzing the nature (social cost, change in investment timing) of the distortions from various tariff policies. Moreover, he does not consider the possibility of a two-part tariff to mitigate those distortions. This paper analyzes those possibilities.

We use a methodology for analyzing entry and exit options from Dixit et al. (1999); Sødal (2006) to analyze this model.

2 The Model

We study a market in which one or two producers produce a product that sells in a competitive market.

Demand is proportional to a stochastic variable X that follows the process

$$dX = \mu X\,dt + \sigma X\,dz\,. \tag{1}$$

We assume this risk is unsystematic, so the true probability process and the risk-neutral probability process are the same.

The Builder has annual sales of $q_B = \alpha_B X$, and the Seeker has sales of $q_S = \alpha_S X$, where $0 < \alpha_S \le \alpha_B$ are constants.. Thus, the two face the same global shocks, but the Builder is a larger entity.

A facilities provider or Builder, B, has real options to build and to abandon a facility of capacity Q units of annual production for a capital cost of K. Capacity is lumpy, and the Builder cannot build anything other than exactly Q units of capacity. We suppose there is facilities access legislation under which another entity, S can seek access to a fraction $f \in (0,1)$ of this production capacity to make its own product. The access Seeker must pay the Builder a two-part tariff consisting of a fixed entry fee of $k_S f Q$, where k_S is the tariff rate, and an annual take-or-pay capacity charge of TfQ, where T is the annual toll rate.[1] This toll rate is in addition to any variable operating costs for the facility, which we take as an offset to selling price. That is, we can embed any variable costs in the net selling price P_B, P_S is net of these variable costs. The Builder and the access Seeker face the same product selling price, and they are price takers, but the Seeker is assumed to have operating costs that are at least as large as the Builder, so $P_S \le P_B$.

To summarize the access policy, the access Seeker must nominate or reserve capacity f (and it must be excess to the then-current needs of the Builder) for a fixed fee that is paid at the time it claims the capacity. Thereafter, the Seeker must pay an annual toll fee proportional to the capacity used, until it abandons the use of capacity. When

[1] This includes the common single-part tariff structure, when $k_S = 0$.

it abandons the project, it does not receive a refund from the Builder (there is no way to require the Builder to maintain liquidity as a refund to the Seeker). But, it does gain relief from having to pay the annual capacity charge.

If the Builder is using $q_B \in [0, Q]$ units of capacity at the time the Seeker nominates the capacity, then, since the Seeker can only use excess capacity, we must have $f \in (0, 1 - q_B/Q)$. In particular, if the Seeker waits until the Builder is using all capacity, it is not allowed to enter.[2]

We first analyze the access Seeker's problem to determine rational behaviour and optimal value. Given this behaviour, we study the Builder's problem to determine optimal behaviour and value. Behaviours of the Builder and Seeker are characterized by demand thresholds or triggers that they set optimally.

The Builder enters the market (builds) when the demand shock first rises above an endogenous trigger threshold X_{BB}. The Seeker enters the market at a threshold X_{SE} (which must be higher because the Seeker needs the facilities that the Builder constructs). At some higher threshold, $X_{FB} = \frac{Q(1-f)}{\alpha_B}$, the Builder is using full capacity, and does not have stochastic production until demand falls below this threshold. At another point, which may be higher or lower, $X_{SF} = \frac{fQ}{\alpha_S}$, the Seeker is using its full capacity and does not have stochastic production until demand falls below this threshold.

On the other hand, at some threshold $X_{SA} < X_{SE}$, the access Seeker will abandon the market, with the only benefit being the relief from having to pay the annual access tariff. At a lower threshold $S_{BA} < \min\{X_{SA}, X_{BB}\}$, the builder also abandons the market to receive a salvage value $S_B \in [0, K)$. If $S_B = 0$, the Builder will never abandon, since it pays no fixed costs. If there are fixed costs of production, their present value can be included in the capital cost K, salvage value S_B and the tariff T.

We assume the riskless interest rate is r.

3 The Access Seeker's Problem

The access Seeker has decision variables f, X_{SE}, X_{SA} and it has different value functions when it is

- waiting for development, W, when $X < X_{SE}$, with value $V_{S,W}(X; f, X_{SE}, X_{SA})$
- producing at the rate $\alpha_S X$ when production has been started and $X_{SA} < X \leq X_{SF} \equiv \frac{fQ}{\alpha_S}$, with value $V_{S,P}(X; f, X_{SE}, X_{SA})$
- producing at the capped rate fQ when $X \geq X_{SF}$, with value $V_{S,SF}(X; f, X_{SE}, X_{SA})$

[2] If the Seeker enters at the same time as the Builder enters, it could, in principle, form a joint venture with the Builder, sharing the capital costs in the proportion f and foregoing the annual operating tarrif, so that $T = 0$.

- abandoned, A, and cannot re-open, with value $V_{S,A}(X; f, X_{SE}, X_{SA}) = 0$.

The problem is time invariant, so the fundamental partial differential equation for valuation is an ordinary differential equation. There is an ODE for each of these 3 regions, and they are bound together by value-matching conditions at their end-points, and they are optimized by the smooth pasting conditions at the boundaries. This is the traditional solution to the problem, as in Dixit (1989), for example. However, we can simplify the analysis using the discount factor approach of Dixit et al. (1999); Sødal (2006). We will discuss both approaches.

3.1 Access Seeker is Waiting for Development

$$\frac{\sigma^2 X^2}{2}\frac{d^2 V_{S,W}}{dX^2} + \mu X \frac{dV_{S,W}}{dX} = rV_{S,W} \tag{2}$$

The value-matching condition is from closed to open, where it must pay the up-front tariff $k_S f Q$:

$$V_{S,W}(X_{SE}; f, X_{SE}, X_{SA}) = V_{S,P}(X_{SE}; f, X_{SE}, X_{SA}) - k_S f Q \tag{3}$$

It also must satisfy the feasibility constraint

$$f \leq 1 - \frac{q}{Q} \tag{4}$$

where $q = \alpha_B X_{SE}$ is the production of the Builder at the time it enters.

3.2 Access Seeker is Operating Below its Nominated Capacity fQ

$$\frac{\sigma^2 X^2}{2}\frac{d^2 V_{S,P}}{dX^2} + \mu X \frac{dV_{S,P}}{dX} + \alpha_B X P - T f Q = rV_{S,P} \tag{5}$$

One value-matching condition is from the transition of producing freely to producing at the nominated capacity:

$$V_{S,P}(X_{SF}; f, X_{SF}, X_{SA}) = V_{S,SF}(X_{SF}; f, X_{SE}, X_{SA}) \tag{6}$$

The other value-matching condition is from the transition of producing freely to abandonment:

$$V_{S,P}(X_{SA}; f, X_{SF}, X_{SA}) = 0. \tag{7}$$

3.3 Access Seeker is Constrained by its Nominated Capacity fQ

$$\frac{\sigma^2 X^2}{2}\frac{d^2 V_{S,SF}}{dX^2} + \mu X \frac{dV_{S,SF}}{dX} + QP - T f Q = rV_{S,SF} \tag{8}$$

The value-matching condition is the same as equation (6).

4 The Discount Factor Solution

Dixit et al. (1999); Sødal (2006) develop a nice procedure for solving these switching problems. The procedure extends the notion that we can derive the present value (PV) of an annuity by taking the present value of a perpetuity $1/r$ and subtracting the present value of a delayed perpetuity e^{rt}/r representing the present value of the cash flows lost when the payments are stopped at time t. The terminal point of the annuity is a boundary where the cash flow stream transitions to a new value.

The discount factor approach involves finding the expected PV factor for the hitting time or first transition time to the boundary. We can take the value of perpetual cash flows, ignoring the boundary, and subtract the expected hitting-time PV of the change in value when the process hits the boundary. The change in value may require knowing the value if the stochastic process returns to the original boundary, but this only results in a linear system of equations for the two boundary values.

The expected PV factor is a value that is computed from the fundamental PDE (ODE in our case) for valuation, coupled with a terminal value of 1 at the boundary. That is, consider the stochastic process (1) for X. (We have assumed that this is both the true and the risk-neutral process for X, and what we need is the risk-neutral process.) Let $D(X_1; X_2)$ be the expected PV factor for the random transition time from $X = X_1$ to a fixed value X_2. This is the value at a point where $X = X_1$ of a security $D(X; X^\star)$ that pays nothing prior to hitting the boundary, and pays 1 when the boundary is first hit. Thus, B satisfies the fundamental ODE for valuation:

$$\frac{\sigma^2 D^2}{2}\frac{d^2 D}{dX^2} + \mu D \frac{dD}{dX} = rD \tag{9}$$

with the terminal condition

$$D(X^\star; X^\star) = 1 \tag{10}$$

The general solution to the ODE (9) is

$$D(X) = A_1 X^{\gamma_1} + A_2 X^{\gamma_2} \tag{11}$$

where

$$\gamma_1 = \frac{1}{2} - \frac{\mu}{\sigma^2} + \sqrt{\left(\frac{1}{2} - \frac{\mu}{\sigma^2}\right)^2 + \frac{2r}{\sigma^2}} \tag{12}$$

$$\gamma_2 = \frac{1}{2} - \frac{\mu}{\sigma^2} - \sqrt{\left(\frac{1}{2} - \frac{\mu}{\sigma^2}\right)^2 + \frac{2r}{\sigma^2}} \tag{13}$$

Following the usual limiting arguments for $X \to 0$ and $X \to \infty$, we can separate the cases where X has to increase to its boundary ($A_2 = 0$) and X has to decrease to its bundary ($A_1 = 0$), we can determine the other A_i by the appropriate value matching condition, and conclude that the expected discount factor to the boundary is

$$D(X; X^\star) = \begin{cases} \left(\frac{X}{X^\star}\right)^{\gamma_1} & \text{if } X < X^\star, \\ \left(\frac{X}{X^\star}\right)^{\gamma_2} & \text{if } X \geq X^\star. \end{cases} \tag{14}$$

This solution works directly when there is only one transition boundary out of the region. When the region has two boundaries, (Sødal, 2006, Section 4) shows how to compute the conditional expected PV to the two transition boundaries, and combine them to solve the the value for the problem with upper and lower boundaries, corresponding to entry and exit. To consider this two-boundary problem, let $D(X; X_1, X_2)$ be the expected discount factor to hitting the boundary X_1 first before hitting X_2. That is, it is the value of a security that pays 1 at the first passage time for the boundary X_1, provided that the boundary X_2 has not yet been reached. We can have $X_1 < X_2$ or $X_2 < X_1$. The ODE for the value of D is still equation (9), so the general solution is still equation (11), but we have new boundary conditions:

$$\begin{aligned} D(X_1; X_1, X_2) &= 1 \\ D(X_2; X_1, X_2) &= 0. \end{aligned}$$

The conditions says that the discount factor to the boundary is 1 if we start at that boundary and 0 if we start at the other boundary. Substituting these into equation (11) gives two equations for A_1, A_2:

$$\begin{aligned} A_1 X_1^{\gamma_1} + A_2 X_1^{\gamma_2} &= 1 \\ A_1 X_2^{\gamma_1} + A_2 X_2^{\gamma_2} &= 0. \end{aligned}$$

After some elementary linear algebra, we find:

$$D(X; X_1, X_2) = \frac{X^{\gamma_1} - X_2^{(\gamma_1 - \gamma_2)} X^{\gamma_2}}{X_1^{\gamma_1} - X_2^{(\gamma_1 - \gamma_2)} X_1^{\gamma_2}}. \tag{15}$$

We can also verify that it provides sensible discount factors if the process X starts between the two boundaries: $0 \leq D(X; X_1, X_2) \leq 1$ if and only if $\min\{X_1, X_2\} \leq X \leq \max\{X_1, X_2\}$. Also, when $X_2 \to \infty$, this approaches the standard abandonment option with threshold X_1, and when $X_2 \to 0$, it becomes the standard development option with threshold X_1.

We now use these expected discount factors to solve the individual sub-problems of Section 3.

4.1 Access Seeker is Waiting for Development

We use the expected discount factor to the development boundary and the change in value at the boundary. That is, prior to development, the Seeker has a perpetual stream of a 0 cash flow. But, at the time of development ($X = X_{SE}$), it exchanges this for a perpetuity in sales net of tariffs, worth $\alpha_S X_{SE} P_S/(r-\mu) - TFQ/r$. The perpetuity is offset by the expected PV of the value changes at the boundaries accessible after development. One is the expected PV of the value lost if X hits the cap X_{SF}, which is $D(S_{SE}; X_{SF}, X_{SA})\left(\frac{\alpha_S X_{SF} P_S}{(r-\mu)} - \frac{TfQ}{r} - V_{S,SF}(X_{SF}; f, X_{SE}, X_{SA}\right)$. Note that we must use the conditional expected PV factor, since this is the case where X hits the capacity boundary before it hits the abandoment boundary. The other is the loss of perpetual revenue at abandonment, again with a conditional expected PV factor, which is $D(S_{SE}; X_{SA}, X_{SF})\left(\frac{\alpha_S X_{SE} P_S}{(r-\mu)}\right)$. Note that this implicitly means that the gain at abandonment is the relief from having to make future tariff expenditures. Thus:

$$
\begin{aligned}
V_{S,W}(X_{SE}; f, X_{SE}, X_{SA}) = {} & \frac{\alpha_S X_{SE} P_S}{(r-\mu)} - \frac{TfQ}{r} - k_S f Q \\
& - D(X_{SE}; X_{SF}, X_{SA})\left(\frac{\alpha_S X_{SF} P_S}{(r-\mu)} - \frac{TfQ}{r} - V_{S,SF}(X_{SF}; f, X_{SE}, X_{SA})\right) \\
& - D(X_{SE}; X_{SA}, X_{SF})\left(\frac{\alpha_S X_{SE} P_S}{(r-\mu)}\right) \text{ for } X \le X_{SE}. \qquad (16)
\end{aligned}
$$

Prior to development ($X < X_{SE}$), the value is the expected PV of the value at development:

$$V_{S,W}(X; f, X_{SE}, X_{SA}) = D(X, X_{SE}) V_{S,W}(X_{SE}; f, X_{SE}, X_{SA}).$$

4.2 Access Seeker is Operating Below its Nominated Capacity fQ

This case is the extension of subsection 4.1 to the situation where $X \in (X_{SA}, X_{SF})$.

$$
\begin{aligned}
V_{S,W}(X; f, X_{SE}, X_{SA}) = {} & \frac{\alpha_S X P_S}{(r-\mu)} - \frac{TfQ}{r} \\
& - D(X; X_{SF}, X_{SA})\left(\frac{\alpha_S X P_S}{(r-\mu)} - \frac{TfQ}{r} - V_{S,SF}(X_{SF}; f, X_{SE}, X_{SA})\right) \\
& - D(X; X_{SA}, X_{SF})\left(\frac{\alpha_S X P_S}{(r-\mu)}\right) \text{ for } X \in (X_{SA}, X_{SF}). \qquad (17)
\end{aligned}
$$

4.3 Access Seeker is Constrained by its Nominated Capacity fQ

This is a slightly problematic condition, because the stochastic demand shock could, in theory, hit this boundary and reflect back at the boundary. This only happens with probability zero, and the relevant case is that it goes through the boundary before reflecting back. Thus, we will consider the case where $X > X_{SF}$ and then consider the limit as $X \to X_{SF}$, since the value function should be continuous at the boundary. When $X > X_{SF}$, there are two possibilities of subsequent events: either X never hits X_{SF} again, or it does hit X_{SF} again, at which time it reverts to the unconstrained operation. The overall value is the PV of perpetual operation at the boundary minus the expected PV of the value change when it reverts from constrained to unconstrained operation. Thus, for $X > X_{SF}$

$$V_{S,SF}(X; f, X_{SF}, X_{SA}) = \frac{P_S f Q}{r} - D(X; X_{SF}) \left(\frac{P_S f Q}{r} - V_{S,SF}(X_{SF}; f, X_{SE}, X_{SA}) \right) \tag{18}$$

**We need a better characterization of the value in the capped region, since $D(X; X_{SF}) \to 1$ as $X \to 1$, so that, in the limit, (18) does not succeed in identifying the value of $V_{S,SF}(X_{SF}; f, X_{SE}, X_{SA})$. The problem is well-defined, but the solution is a little more subtle than I have here.

One solution is to treat the region $[0, \infty)$ as a perpetual production region where the seeker has lost a set of upside call options on production. We could calculate Black-Scholes call option values for each future date and integrate over dates to get the total value of production lost to the cap. Then, this value would be adjusted by the probability of going to the abandonment boundary. The solution would be numerical.

4.4 Access Seeker Optimization

The valuations above are conditional on the decision parameters f, X_{SE}, X_{SA} for the access Seeker. We can numerically optimize the value prior to development over these parameters, and we have the policy and value for the access seeker.

5 The Facilities Builder Problem

The solution of the facilities Builder will proceed in a similar fashion, where the Builder must determine the threshold at which to build and to abandon. Given its decisions, the access Seeker will optimize its decisions, so the Builder will rationally anticipate this optimal response in determining its own policy. We can numerically solve for the Builders decision policy and value.

6 Analysis

Given the solution to the Seeker and Builder problem, we can proceed to analyze how the solution varies with the two-part tariff policy. We can see how the Builder's decisions will change for different tariff policies, identifying how the tariff decisions cause it to produce early or late. We can also sum the value for the Builder and the Seeker to calculate a social value. This allows us to assess the social value losses from specific tariff policies. For example, we can measure the social loss from a single-part tariff, and we can also measure the social loss when that tariff reimburses out-off pocket construction costs only.

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